

THEORETICAL ANALYSIS OF NONLINEAR OPTICAL WAVEGUIDE SENSORS: TE CASE

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ABSTRACT

In the present paper, an analytical theory will be investigated for studying three-layer slab waveguide for sensor applications. The sensor is a proposed nonlinear optical waveguide including a linear thin film with thickness h and dielectric constant ϵ_f that is surrounded by a substrate with nonlinear permittivity ϵ_1 and a cover with nonlinear permittivity ϵ_2 . Nonlinear environments are of Kerr type. One of the best quantities studied in the research is sensor sensitivity, that is determined with change in effective refractive index N for changing in refractive index of cover medium n_c . Status of obtaining max sensitivity for nonlinear waveguide sensors TE has been investigated and compared with linear sensors. Guide layer has been chosen as Si_3N_4 and GaN and we have applied silica as cover and substrate environment with some percent impurity. Our proposed sensor with sensitivity of $S_h = 0.2798$ increases up to 105.433% and thickness of sensor has decrease of $h=105$ nm up to 41.666% compared to previous proposed sensors of researchers.

KEYWORDS: Waveguide Sensor, Sensitivity, Refractive Index, Cover, Substrate

INTRODUCTION

In the recent two decades, optical waveguide sensors have attracted much attention. In the optical fiber changes process, various investigations have been done that its concentrated has been on suitable design of sensors. As offshoot of these observations, new ideas using for the design of sensor system that had been led to fiber-based sensor devices and parts (Yu, et al 2002; Gholamzadeh, et al 2008; Horvath, et al 2003; Taya, et al 2011a; Abadla, et al 2004a).

Optical sensors used in modification of chemical measurements to optical properties such as intensity, phase, frequency,.... Chemical sensors considered for immunity in electromagnetic interference, mechanical resistance, small size, ... and for this reason, it can be used in hazardous environments (Niu, et al 2011; El-Agez, et al 2011; Taya, et al 2010; Taya, et al 2012).

In the past years, optical sensors have been used in different cases for various conduction such as biology, biochemical and biosensors (Benaissa, et al 1998; Qing, et al 1999; Abadla, et al 2004b; Abadla, et al 2003; Veldhuis, et al 2000). With the use of these sensors physical quantities can be measured such as electric current, magnetic field, pressure, heat, displacement, pollution of sea waters, liquid level, gamma and X rays.

Its further application in medical can be to detect the concentrations of certain chemicals in blood, pharmaceutical applications, cancer tumors dosimeter, identifying of deficiencies within the body, laser surgery, dentistry application, liquid measurement and blood.

The essential part of an optical waveguide sensor consists of a high refractive index wave guiding film sandwiched between a substrate and a covering. The waveguide sensor structure can be divided into two configurations. The first configuration is called a normal symmetry sensor which the substrate with a refractive index higher than that of the cover. The second configuration is the substrate with a refractive index less than that of the cover. The waveguide sensor is called a reverse symmetry waveguide. In general, the reverse symmetric waveguide sensors have relatively higher sensitivity (Parriaux, et al 2000; Veldhuis, et al 1999; Skivesen, et al 2003; Horvath, et al 2003; Taya, et al 2011b; Schmitt, et al 2010).

Effective refractive index (N) is one of the most important factors in the field of optical sensing that is obtained by equation $N = c/v_{ph}$, where c is the speed of light in vacuum and v_{ph} is the phase velocity. The change in effective refractive index for each cover evanescent wave induced interactive changes. This effect is the basis of optical sensors.

When a light beam with alpha angle was injected in an end of waveguide in film part and the light is guided within film by total internal reflections in boundaries of film-cover and film-substrate in around of angle θ and exited from the other end of the waveguide the intensity of which was measured by detector (Abadla, et al 2004; Brioude, et al 2000; Skivesen, 2005; Taya, et al 2012; Taya, et al 2013; Taya, et al 2011c). Evanescent wave formed in each point of integrated internal reflection. A small part of evanescent wave distributed into the covering environment in comparable space with guidance light wavelength. This Evanescent wave is important in sensitivity operating. When cover is placed in the evanescent field, change is observed in absorption or phase shift the light propagating in waveguide. This observed change is the sign of concentration or cover refractive index. This displacement between cover and evanescent field can be observed with spectroscopic techniques.

THEORY

Here, we are primarily interested in examining the behavior of nonlinear slab waveguides in sensing applications. In the present study the p-polarized waves propagate through x axes are considered.

Main part of a nonlinear waveguide sensor consists of a linear thin film of thickness h and dielectric constant ϵ_f . This film is sandwiched between a nonlinear substrate and a nonlinear covering layer. Where covering of nonlinear permittivity (ϵ_1) and substrate of nonlinear permittivity (ϵ_2) are given by:

$$\epsilon_1 = \epsilon_c + \alpha_c |E_{y1}|^2. \quad (1)$$

$$\epsilon_2 = \epsilon_s + \alpha_s |E_{y3}|^2. \quad (2)$$

Where α_c and α_s are the nonlinearity coefficients of covering and substrate, respectively, ϵ_c and ϵ_s are the linear parts of the permittivities and E_{y1} and E_{y3} are the TE fields in covering and substrate, respectively. The electric field in each layer has the following solution for TE modes for $\alpha_c > 0$ and $\alpha_s > 0$:

$$E_{y1} = \sqrt{\frac{2}{\alpha_c}} q_c \operatorname{sech}(k_0 q_c (z - z_c)), \quad z > h \quad (3)$$

$$E_{y2} = A \cos(k_0 q_f z) + B \sin(k_0 q_f z), \quad 0 < z < h \quad (4)$$

$$E_{y3} = \sqrt{\frac{2}{\alpha_s}} q_s \operatorname{sech}(k_0 q_s (z_s - z)), \quad z < 0. \quad (5)$$

Where $q_c = \sqrt{N^2 - \epsilon_c}$, $q_f = \sqrt{\epsilon_f - N^2}$, $q_s = \sqrt{N^2 - \epsilon_s}$, N is the effective refractive index, constants A and B represent the amplitude of the waves, and z_c and z_s are constants related to the field distribution in the covering medium and the substrate, respectively.

Continuity of E_y and $\frac{\partial E_y}{\partial z}$ gives the dispersion relation

$$\tan(k_0 q_f h) = \frac{q_c q_f \tanh C_c + q_s q_f \tanh C_s}{q_f^2 - q_c q_s \tanh C_s \tanh C_c}. \quad (6)$$

Where k_0 is the free space wave number, $C_c = k_0 q_c (h - z_c)$, and $C_s = k_0 q_s z_s$ which are called the covering film interface nonlinearity and the substrate film interface nonlinearity respectively.

Where a_c and a_s are two asymmetry parameters and X_c and X_s are two normalized variables, where a_c , a_s , X_c , X_s are given by:

$$a_s = \frac{\epsilon_s}{\epsilon_f}, a_c = \frac{\epsilon_c}{\epsilon_f}, X_s = \frac{q_s}{q_f}, X_c = \frac{q_c}{q_f}. \quad (7)$$

X_c and X_s are linked by:

$$X_c^2 = \frac{(1-a_c)(1+X_s^2)}{(1-a_s)} - 1. \quad (8)$$

The effective refractive index can be written in terms of a_s and X_s as:

$$N = \sqrt{\epsilon_f} \sqrt{\frac{a_s + X_s^2}{1 + X_s^2}}. \quad (9)$$

In terms of X_c and X_s Eq. (6) can be written as

$$k_0 q_f h - \arctan(X_c \tanh C_c) - \arctan(X_s \tanh C_s) - m\pi = 0. \quad (10)$$

Where $m = 0, 1, \dots$ is the mode order.

From the dispersion relation given by Eq. (10), we derive the sensor sensitivity S_h , i.e., the basic sensing principle of the planar dielectric waveguide sensor is to measure changes in N due to changes in n_c . Differentiating Eq. (10) with respect to N and calculating S_h as $\left(\frac{\partial n_c}{\partial N}\right)^{-1}$ we obtain:

$$S_h = \frac{\sqrt{a_c} \sqrt{1+X_c^2} (H_c + \tanh C_c)}{X_c \sqrt{a_c + X_c^2} (1+X_c^2 \tanh^2 C_c) (A_{TE} + G_s + G_c)} \quad (11)$$

Where

$$H_c = k_0 (h - z_c) X_c \sqrt{\epsilon_f} \sqrt{\frac{1-a_c}{1+X_c^2}} (1 - \tanh^2 C_c). \quad (12)$$

$$G_c = \frac{H_c + \tanh C_c (1+X_c^2)}{X_c (1+X_c^2 \tanh^2 C_c)}. \quad (13)$$

$$G_s = \frac{H_s + \tanh C_s (1 + X_s^2)}{X_s (1 + X_s^2 \tanh^2 C_s)}. \quad (14)$$

$$H_s = k_0(z_s) X_s \sqrt{\epsilon_f} \sqrt{\frac{1 - a_s}{1 + X_s^2}} (1 - \tanh^2 C_s). \quad (15)$$

$$A_{TE} = \arctan (X_s \tanh C_s) + \arctan (X_c \tanh C_c) + m\pi. \quad (16)$$

The sensitivity of the proposed nonlinear sensor is given by Eq. (11) which strongly depends on the waveguide structure.

One of the most important quantities is the power flow in different layers of the slab waveguide. The sensitivity of the sensor is critically dependent on the fraction of total power propagating in the covering medium. The energy flux per unit length is given by

$$P = \int_{-\infty}^{+\infty} E \times H dz = P_s + P_f + P_c. \quad (17)$$

Where P_s is the power flow in nonlinear substrate and P_f is the power flow in linear film.

The fraction of total power flowing in the nonlinear covering is:

$$\frac{P_c}{P_{total}} = \frac{\frac{X_c j_1}{\alpha_c}}{\frac{X_c j_1}{\alpha_c} + \frac{X_s^2 \operatorname{sech}^2 C_s r + X_s j_2}{2\alpha_s}}. \quad (18)$$

Where

$$r = k_0 q_f h x_+ + \frac{1}{2} \sin(2k_0 q_f h) x_- + X_s \tanh C_s j_3. \quad (19)$$

$$j_1 = 1 - \tanh C_c. \quad (20)$$

$$j_2 = 1 - \tanh C_s. \quad (21)$$

$$j_3 = 1 - \cos(2k_0 q_f h). \quad (22)$$

$$x_- = 1 - X_s^2 \tanh^2 C_s. \quad (23)$$

$$x_+ = 1 + X_s^2 \tanh^2 C_s. \quad (24)$$

REPRESENTATION AND DISCUSSIONS

In our calculations, we will assume the guiding layer to be Si_3N_4 ($n_f = 2$) and GaN ($n_f = 2.34949$), the free space wavelength will be $\lambda = 1550$ nm, $\tanh C_c = 0.6$ and $\tanh C_s = 0.7$. Only the fundamental mode ($m = 0$) will be considered since it has the highest sensitivity.

Substratum and covering medium are considered silica with different impurity percent. It is necessary to mention that silica's linear and nonlinear dielectric constant is respectively $\epsilon = 2.13$ and $\alpha = 7.04 \times 10^{-20}$.

In figure (1), proposed nonlinear sensor sensitivity is drawn based on guidance layer thickness h that conventional symmetric waveguide (i.e. $n_s > n_c$) is considered. Covering medium for this diagram is silica with $\epsilon_c = 2.13$ and substrate environment has been regarded as silica with three different kinds of impurity which are 0.09%, 0.9% and 10% dielectric constant impurity consist of $\epsilon_s = 2.311917, 2.14917$ and 2.343 and diagrams of each one are drawn

separately and then they are compared. As it is seen in the figure, substrate with $\epsilon_s = 2.311917$ has the best sensitivity and minimum thickness of guidance layer h . It should be noted that $n_f = 2$ has been considered and then diagram for $n_f = 2.34949$ is drawn in figure (2) and both figures are compared.

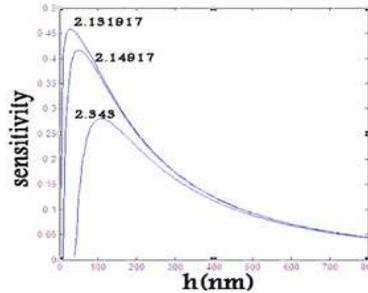


Figure 1: Sensitivity with the Film Thickness for $n_f = 2$, $\epsilon_c = 2.13$, $\epsilon_s = 2.14917$, $\epsilon_s = 2.131917$, $\epsilon_s = 2.343$

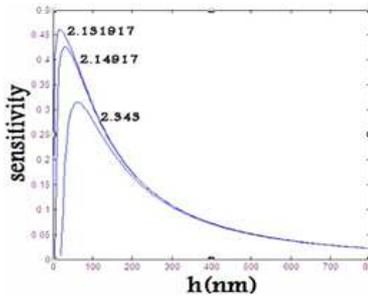


Figure 2: Sensitivity with the Film Thickness for $n_f = 2.34949$, $\epsilon_c = 2.13$, $\epsilon_s = 2.14917$, $\epsilon_s = 2.131917$, $\epsilon_s = 2.343$

The sensitivity of the proposed sensor was plotted with the film thickness for the different asymmetry parameters ϵ_s in Figure 1 and 2. The figures show that the sensitivity decreases with increasing ϵ_s . This is a normal behavior. The sensitivity operation is depending on the evanescent optical field extending from the thin guiding film into the covering medium. To obtain high sensitivity, it is essential to get an amount of spreaded optical power in the cover. Therefore, increasing ϵ_c and decreasing ϵ_s is dependent on flowed power fraction in covering.

Comparing figures (1) and (2), it is clear that in $n_f = 2.34949$, sensitivity has been increased and thickness of guidance layer has been decreased. We would like to compare linear and nonlinear sensors that have been shown in figure (3). In linear state, $\tanh C_s = 1$. We can see that sensitivity in nonlinear state is better than the linear one and thickness of sensor guidance layer is thinner in nonlinear.

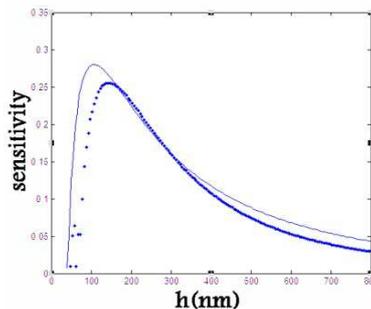


Figure 3: Sensitivity with the Film Thickness for $n_f = 2$, $\epsilon_c = 2.13$, $\epsilon_s = 2.343$ for the Proposed Nonlinear Sensor (Solid Line) and a Linear Sensor (Dotted Line)

One of the important works done in this area is the comparison between suggested sensor diagrams and previous research works that have been shown in figure (4).

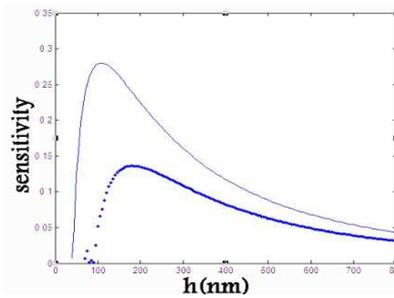


Figure 4: Sensitivity with the Film Thickness for $n_f = 2$, $\epsilon_c = 2.13$, $\epsilon_s = 2/343$ for the Proposed Nonlinear Sensor (Solid Line) and $n_f = 2$, $\epsilon_c = 1.8$, $\epsilon_s = 2.6$ for the Previous Research Works (Dotted Line)

In figure (5), there is a comparison between reverse symmetric sensor ($n_s < n_c$) and normal symmetric sensor. Maximum value of sensitivity near the cut thickness decreases with increase in guidance layer thickness of sensor. This situation is attributed to power considerations. When guidance layer thickness is near cut thickness, effective refractive index N is near covering coefficient n_c , the penetration depth of the evanescent field into the cover medium becomes infinite and total power frequently flows in covering such that guidance layer thickness of reverse symmetric waveguide increases.

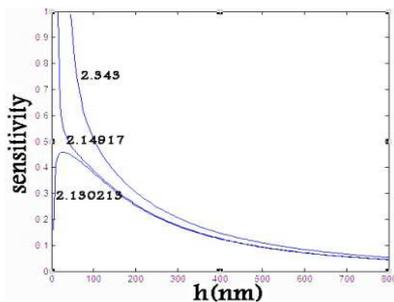


Figure 5: Sensitivity with the Film Thickness for $n_f = 2$, $\epsilon_s = 2.131917$, $\epsilon_c = 2.130213$, $\epsilon_c = 2.14917$, $\epsilon_c = 2.343$

One of the cases that is considered, is sensitivity diagram based on $\tanh C_c$. As shown in figure (6), when $\tanh C_c$ goes toward 1, the sensitivity decreases. This is the prospected subject because number 1 is marker of linear sensors. According to previous diagrams, nonlinear sensors have higher sensitivity comparing to linear sensors. This diagram supports previous diagrams.

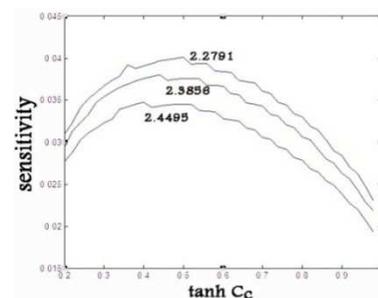


Figure 6: Sensitivity with the Film Thickness for $\epsilon_s = 2.343$, $\epsilon_c = 2.2791$, $\epsilon_c = 2.3856$, $\epsilon_c = 2.4495$, $h = 0.1 \mu\text{m}$

CONCLUSIONS

In this paper, an optical sensor of nonlinear three-layer slab waveguide is analyzed. The Sensitivity of the effective refractive index changes is based on variations of the covering index that can be improved by using nonlinear materials in covering layer and nonlinear materials in substrate. Also, the thickness of guiding layer is a critical parameter for the sensitivity of the optical sensor with the optimum thickness that is equal to the cut-off thickness in case of reverse symmetric waveguides and is above the cut-off thickness in case of normal symmetric waveguides. With use of new materials in our proposed sensor design, we have increased sensitivity of the sensors and have gotten better results. Sensor maximum sensitivity occurs in thinner thicknesses that is a new innovation in the field of sensors design.

Figures (1) and (2) show that impurity percentage has reverse relationship with sensitivity and direct relationship with thickness of guiding layer that means the less impurity percentage results in more increase of sensitivity and lower thickness of guidance layer. Figure (3) suggests that nonlinear diagrams have better sensitivity and lower thickness of guiding layer related to linear sensors and figure (6) confirmed this matter. Figure (5) shows a comparison between reverse and normal symmetric sensors and that reverse symmetric sensors have much better sensitivity. However, their application is impossible in practice. According to figure (4), our proposed sensor with sensitivity of $S_h = 0.2798$ increases up to 105.433% and thickness of proposed sensor to be $h = 105$ nm that shows a type of decreasing about 41.666%. Previous proposed sensors of researchers have sensitivity of $S_h = 0.1363$ and thickness of $h = 180$ nm.

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